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182. Proposed by O. L. CALLICOTT, Gettysburg, S. Dak.

Find the value of $\sqrt[1]{2} \sqrt[3]{2} \sqrt[4]{2} \sqrt[5]{2} \dots \sqrt[1000]{2}$.

I. Solution by W. D. LAMBERT, Washington, D. C.

Let $\frac{1}{2}P$ be the required product.

Then $\log_{10} P = \log_{10} 2(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \dots \frac{1}{1000})$.

By the Euler-Bernoulli formula for reducing summation to integration,

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{x} = v + \log_e x + \frac{1}{2x} - \frac{1}{12x^2} + \frac{1}{120x^4} \dots$$

v is Euler's constant = 0.57721, 56649...

$\therefore \log_{10} P = 0.3010300(0.57721566 + 6.90775528 + 0.00050000 - 0.00000008 \dots)$
 $= 2.253351$.

$\therefore \frac{1}{2}P = 89.603$, the required value.

II. Solution by S. A. COREY, Hiteman, Iowa.

If s is the value sought,

$$\log s = (\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{1000}) \log 2. \quad (1)$$

If $\log(1+x)$ be developed by the formula given in Prize Problem 237, Calculus, we get

$$\begin{aligned} \log(1+x) = & 0 + \frac{x}{2m} \left[\frac{1}{1+x} + 1 + 2 \left(\frac{1}{1+\frac{x}{m}} + \frac{1}{1+\frac{2x}{m}} + \dots + \frac{1}{1+\frac{(m-1)x}{m}} \right) \right] \\ & + \frac{B_1 x^2}{m^2 \cdot 2!} \left[\frac{1}{(1+x)^2} - 1 \right] - \frac{B_2 x^4}{m^4 \cdot 4!} \left[\frac{3!}{(1+x)^4} - 3! \right] + \text{etc.} \end{aligned} \quad (2)$$

If, now, x is an integer and m be taken equal to x , we have, as x approaches ∞ ,

$$\lim_{n \rightarrow \infty} \log n - \left[\frac{1}{2} + \sum_{r=2}^{\infty} \frac{1}{r} \right] = - \left[\frac{B_1}{2} - \frac{B_2}{4} + \frac{B_3}{6} \dots \right] = -b; \quad (3)$$

whence if C be Euler's constant, .577,215,664,901,5..., $b = C - \frac{1}{2}$. Substituting in (2), transposing, adding $\frac{1}{1000}$ to each member, and reducing, we get, by making $(1+x) = 1000$,

$$(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{1000}) = \log 1000 - 1 + \frac{1}{2000} + C - \left[\frac{B_1}{2 \cdot 1000^2} - \frac{B_2}{4 \cdot 1000^4} + \dots \right]$$

$= 6.485,470,860,55$, or $s = 2^{6.485,470,860,55} = 89.602,734.8 \dots$

Also solved by G. B. M. Zerr, J. Scheffer, and A. H. Holmes.